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13. ABSTRACT *(Maximum 200 words)*

This chapter treats C^3 processes from a formal theory viewpoint. The approach is microscopic in nature, using a time-slice model – as opposed, e.g., to the outcome path approach of Petri nets and their generalizations. The usual SHOR paradigm plays the central role in the structuring of nodes, while knowledge-based information also plays a role. These intranodal relations – as well as intranodal relations in the form of signals and communications through medium noise – are combined into a single large-scale formal model. In addition, uncertainty in the form of non-stochastic information, such as through linguistic sources, is taken into account in the data fusion aspect.

The basic model consists of axioms representing the various conditional relations among C^3 SHOR paradigm variables, such as input signals, detection states, manpower, supply levels, damage levels, hypotheses of situations, and decisions and reactions/responses. The choice of the actual functional distributional relations among these variables relative to the axiom constraints can be interpreted as a C^3 design move within a zero-sum game theoretic context. The basic loss function here consists of some pre-chosen moe/mop of the state of "health" of the friendly and adversary C^3 systems. In turn, the health of each side is determined from an averaging procedure over all node states of the individual node state distributions in conditional form following SHOR paradigm signal processing cycles. These node state distributions are obtainable as out-puts of the basic model described above.

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A FORMAL THEORY OF C^3 AND DATA FUSION

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Abstract

This chapter treats C^3 processes from a formal theory viewpoint. The approach is microscopic in nature, using a time-slice model - as opposed, e.g., to the outcome path approach of Petri nets and their generalizations. The usual SHOR paradigm plays the central role in the structuring of nodes, while knowledge-based information also plays a role. These intranodal relations - as well as internodal relations in the form of signals and communications through medium noise - are combined into a single large-scale formal model. In addition, uncertainty in the form of non-stochastic information, such as through linguistic sources, is taken into account in the data fusion aspect.

The basic model consists of axioms representing the various conditional relations among the C^3 SHOR paradigm variables, such as input signals, detection states, manpower, supply levels, damage levels, hypotheses of situations, and decisions and reactions/responses. The choice of the actual functional distributional relations among these variables relative to the axiom constraints can be interpreted as a C^3 design move within a zero-sum game theoretic context. The basic loss function here consists of some pre-chosen moe/mop of the state of "health" of the friendly and adversary C^3 systems. In turn, the health of each side is determined from an averaging procedure over all node states of the individual node state distributions in conditional form following SHOR paradigm signal processing cycles. These node state distributions are obtainable as outputs of the basic model described above.

BACKGROUND ON C^3 ANALYSIS

The history of C^3 analysis as an organized approach to defining the general military problem, and in particular the command and control aspects, goes back several years. For a brief history of approaches based upon the MIT/ONR Workshop on C^3 Systems - for many years the premier academic venue for C^3 analysis - see e.g. Goodman [1]. Despite the large amount of literature produced on C^3 issues- whether it be from the Workshop or other Government or private industry sources - a basic pattern emerges: little attention has been paid to the establishment of an overall C^3 model from a quantitative microscopic or "bottoms-up" point of view. Instead, much of the work has been devoted to either qualitatively-based analysis or to quantitative analysis of bits and pieces of the whole C^3 panarama. This is obvious due to the the great potential complexity involved in attempting to model the entire detailed process. In addition, some papers have been produced approaching C^3 systems from a complete macroscopic or "top-down" viewpoint. Examples of of the first two types of analysis are numerous. Perusing through the last several issues of the Proceedings of the MIT/ONR Workshop and its subsequent successor,the Symposia on C^2 Research, one finds articles on command planning, fire control, tracking and filtering, correlation of multiple targets, surveillance, limited interacting multiple persons decision games, time studies, stochastic control problems, etc. Examples of the last-mentioned type of study are not as plentiful, but include papers on markovian models of C^3 systems relative to attrition and supply, variations of Lawson's macro-thermodynamic analogue, Lanchester's attrition equations and its generalizations, use of general resource allocation principles, and applications of analogues with laws of behavior in economics and other large-scale systems.

Of course, the above-mentioned examples certainly contribute toward the overall understanding of C^3 in general; however, they point up the lack of any attempt to model C^3 from a microscopic approach. It is the thesis here that it is not too early in the development of C^3 as a discipline to make this effort. Among the work directed previously to this goal, mention should be made of Ingber [2,3] and Rubin and Mayk [4,5]. Ingber utilizes the path integral principle from nonlinear nonequilibrium statistical mechanics to attempt a meso-macroscopic C^3 analogue model, while Rubin and Mayk's approach has a more microscopic flavor in extending the Lanchester equations. Finally, the work of Levis

. al. [6-8] should also be noted. This is based upon a partial microscopic model of the SHOR paradigm concerning information throughput and transmittal relative to an overall organizational model. In a sense, this work has influenced the author's thinking more than any other source with respect to modeling of C^3 systems.

OBJECTIVES AND APPROACH

The long-range goal of this work is two-fold:

(1) To show tactical C^3 processes can be reasonably modeled within a game theory context, using a formal system of axioms which capture a minimal number of pertinent relations among the C^3 variables and operators.

(2) To provide an outline for a feasible implementation of this program as an aid in the design of C^3 systems.

For present, we must be content with only the first goal; time will tell whether the second goal can be achieved. In modeling C^3 processes one must be always aware of the tradeoff between the fidelity of theory and the complexity of practical implementations. With this in mind, a C^3 design game is proposed here based upon the outputs of a formal theory for the evolution of node states. This is predicated upon the assumption that a C^3 system as envisioned here is completely identified as a collection of such interacting node states, each operating according to the SHOR paradigm (S=sense, H=hypothesize, O=options available, R=response). Externally, the model can be implemented via standard probability ideas , but internally, two factors involving nonstandard concepts are treated: incorporation of linguistic-based or narrative information and utilization of conditioned information, when the antecedents of the conditioning differ. More details of this will be presented in the following sections.

Before proceeding to the development of the formal theory, a summary of the key ideas in describing and analyzing C^3 systems as viewed here is given in Figures 1-5. Figure 1 illustrates a typical interaction of C^3 nodes. The SHOR paradigm is outlined in Figure 2, with the basic evolution cycle of node signal processing shown in Figure 3. Figure 4 outlines how knowledge flows in general in carrying out a formal theory and, finally, Figure 5 illustrates the decomposition of a C^3 node state into its proper and knowledge parts.

Qualitative Aspects

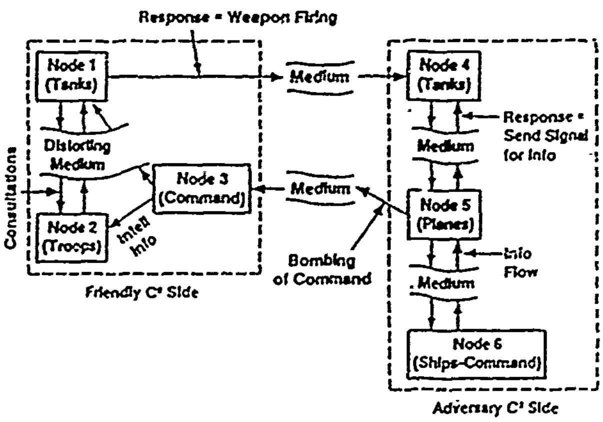


Figure 1. External Dynamics of C³ Processes: Simplified

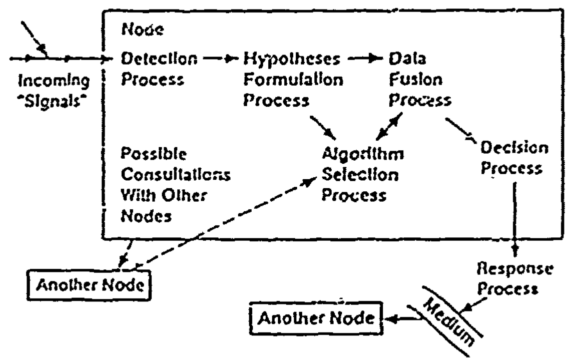


Figure 2. Internal Dynamics of C³ Processes: Simplified

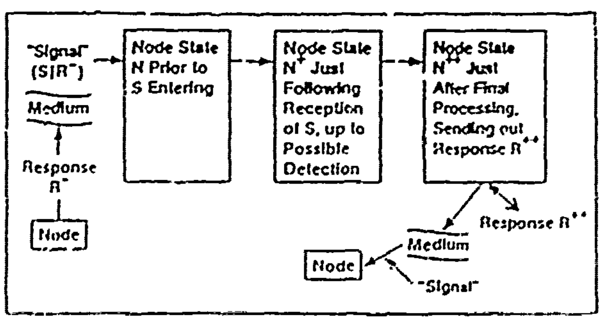


Figure 3. Basic Evolution Cycle of a Node Due to "Signal" Processing and Response.

Quantitative Aspects

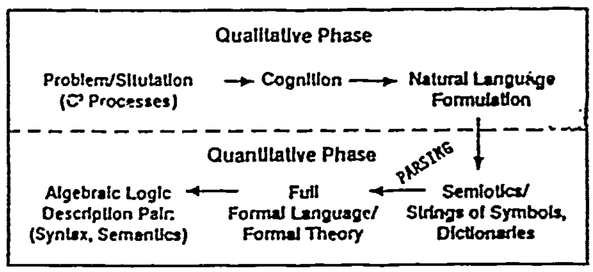
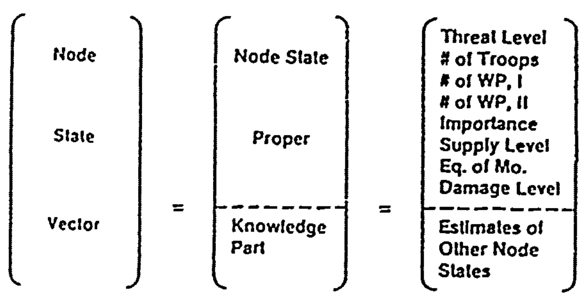


Figure 4. Knowledge Flow in Describing Situations



NODE = (NODE STATE , NODE STRUCTURE)

Figure 5. Components of C^3 Node States

RATIONALE FOR USE OF FORMAL THEORY

Basically, all scientific disciplines are concerned with developing theories as comprehensive as possible. This is both to explain past empirically obtained data and to predict as accurately as possible likely future performance or behavior. Given sufficient specialization and localization, these goals have been realized to varying degrees of success in many areas comprising the "hard" sciences. These include, for example: natural/physical phenomena studied in Mechanics, Bilogy, and Chemistry; the more abstract-rooted, but related fields of Statistical Communications and Information, and the yet more abstract general fields of Mathematics and Logic. On the other hand, much less success has been achieved in developing explanatory theories for the "soft" sciences related to human thought and relations, including Natural Language, Cognition, Psychology, Sociology, and Law.

At the outset, it must be pointed out that any attempt at describing systematically C^3 ought to span both soft and hard sciences. This is due to the interdependencies of the following three facors:

a. necessary physical actions and effects (possibly deadly) involved in moving about men and supplies relative to the execution of weapons and resulting damage given and received

b. decision-making schemes used in carrying out all aspects of part a

c. information content present upon which part b operates (in this case both sensor-oriented (stochastic) and human-oriented (linguistic) information may be present)

In synthesizing the above-mentioned concepts into a coherent whole, it is reasonable to attempt a full formal approach, drawing from previous more localized situation-specific analyses of C^3 systems. Such a comprehensive formal theory of C^3 can help in the long run to place it more within the realm of science rather than just art and empiricism. *Most importantly, such a framework is relatively easily amenable to changes - such as for finer tuning or modifications of [illegible] - when deemed necessary, and, as well, exhibits the basic logical relations and algebraic structure for all the variables.*

Examples of formal theory abound. To illustrate this point, see e.g. the work of Woodger [9] in Biology, Carnap [10] for aspects of Sociology and Law, and more recently, in attempting to axiomatize Quantum Mechanics [11]. Also, by using a formal theory C^3 systems can be analyzed from a more universal mathematical-logical viewpoint. In particular, the newly-developed calculus of conditional events [12,13] becomes readily applicable to more consistent modeling of combination of conditional evidence, as part of data fusion.

The theory outlined in the subsequent sections is based upon a distillation of the work found in [14-18].

C^3 VARIABLES

In building the formal theory, one must first scope out the relevant variables describing the system. Generally, these variables are indicated by the end of the alphabet letters as X,Y,Z. Particular variables are denoted by other letters such as R denoting response of a node, N for the entire node state, and ALG for the algorithm selected for a given node following "signal" reception (the quotes later to be explained), etc. Two specifically designated variables are actually constants: Ω for the universal or always true event or action and 0 for the null or always false event or action.

Each variable, where necessary, indicates through appropriate subscripting or superscripting the time, hostile vs. friendly status of C^3 system, as well as the identification number. In addition, each variable X has associated with it a natural domain of values that the variable can achieve. Call this dom(X). Depending on the nature of the variable X, dom(X) consists of usually a collection of subsets of a given set, or in particular, of a collection of singleton "points" making up the parent set dom(X).

It should also be noted that all axioms involving variables can be converted to corresponding ones with any domain value substituted for the corresponding variable. For convenience, variables can be divided into two basic types - intranodal and internodal - with further subdivisions where warranted.

INTRANODAL VARIABLES

These designated variables describe the functioning of a typical C^3 decision node. Three subdivisions arise: node state proper, knowledge aspect, and node structure.

(1) N denotes the ensemble of node state proper variables for a typical node. Some examples include: TRP, the number of troops present, EQ, the true equations of motion of the entire node, such as straight line constant velocity, second degree motion in a parabolic path, circular constant tangential motion, etc. Also,

WP6 indicates number of weapons of type 6 present in the node and DAM damage level to the node so far. Thus, one can write typically

$$N = (..,TRP,EQ,..,WP6,..,DAM,..) , \tag{a}$$

filling in the appropriate varibles.

(2) K denotes the collection of knowledge-related variables for the node of interest. Generally, this is taken here to be the estimates of tne variables belonging to all other nodes, friendly or hostile. In many cases, this will be vacuous from lack of pertinent information. Thus, e.g., one might have $\hat{WP6}_{i,j}$ indicating node i's estimate of WP6 relative to node j. A typical example of K can be

$$K_3 = (..,\hat{EQ}_{3,7} ,..,\hat{WP4}_{3,2} ,..) . \tag{b}$$

(3) T denotes the collection of variables describing the actual functioning of the node. These include DET, detection, HYP, hypotheses formulation, ALG, algorithm selection, FUS, data fusion, and DEC, decision, all based upon incoming "signal" S. The quotes about S refer to the fact that S could be a signal in the classical sense or an incoming weapon about to explode, or any other physical or sensory interaction between nodes. Thus, one could write

$$T = (DET,HYP,ALG,FUS,DEC,..) \tag{c}$$

INTERNODAL VARIABLES

The second type of C^3 variable is the internodal or between-node type. These variables describe the factors present that affect and relate one decision node with another. These include R, the node response following all data/"signal" processing of "signal" S and S itself.

The fundamental relationship between an outgoing node response becoming eventually a "signal" relative to another node or nodes is determined by the intervening environment or medium which can distort and/or produce "additive" (in some algebraic sense such as ordinary arithmetic addition or multiplication) noise to the original response. Symbolically, one has the general regression relation

$$S = G(R) \oplus Q . \tag{d}$$

where internodal variable G is actually a numerically valued (vector or scalar) function which is possibly nonlinear in R and Q represents addtive error. (Other relations among internodal and intranodal variables will be considered in the next sections.) Thus, the internodal variables can be displayed as, say,

$$J = (S,R,G,Q) \quad , \tag{e}$$

with, of course, suitable time and node identifier indices.

EXAMPLES OF DOMAINS OF VARIABLES AND TRANSFORMS OF LINGUISTIC INFORMATION TO STOCHASTIC

Some examples of domains are

$$\mathrm{dom}(TRP) = \{0,1,2,3,\ldots,6000\} \quad , \mathrm{dom}(EQ) = \{e(s,v,a)\colon s\epsilon S_0, v\epsilon V_0, a\epsilon A_0\} \ ,$$

where $e(s,v,a)$ indicates constant acceleration two-dimensional motion with initial position s, initial velocity v, and constant acceleration (possibly 0) a, where S_0, V_0, A_0 are suitably chosen sets of 2 by 1 real vectors.

Some additional examples worth noting:

$$\mathrm{dom}(ALG) = \{ALG1, ALG2,\ldots, ALG31\} \ ,$$

where ALG1 is a piece-wise linear Kalman filter, ALG2 is an alpha-beta gilter,..., ALG17 is a hypotheses tester which assumes the general linear regression model, ALG18 is a hypotheses tester based upon AI procedures,... .

$$\mathrm{dom}(DET) = \{\text{no detect}, \text{detect}\} \quad , \quad S = \{S_1, S_2, \ldots, S_{23}\} \ ,$$

where each S_i is a linguistic or stochastic variable. For examples of linguistic variables:

S_1 = "ship appears short- maybe under 300 feet long"

S_2 = "ship appears to be of medium length - maybe in the neighborhood of 200-400 feet long"

S_3 = "ship appears to be very wide and in fog a reddish flag was spotted" .

On the other hand, $S_4, S_5, \ldots, S_{23}$ can represent stochastic variables, such as

$$S_5 = \text{"ship}_1 \text{ is 4.8 miles from ship}_2\text{"} ,$$

where the above outcome is assumed to come from an exponential distribution with parameters $\lambda = 0.7$ miles and $\sigma = 2.1$ miles, so that S_5 represents the outcome of a random variable with a well-defined distribution which is known.

In the case of S_3 and other linguistic-based descriptions, one can utilize a technique (see [19]) which converts first the linguistic description to a fuzzy set or possibilistic form and then to a random set structure, or equivalently, a cdf. For example, S_3 can be stated as

$$S_3 = (\text{ht(ship)} \ \varepsilon \ \text{very(long)}) \cdot (\text{col(flag)} \ \varepsilon \ \text{reddish} \mid \text{weather} \ \varepsilon \ \text{fog}) ,$$

where the symbols ε and $\cdot$ above refer to formal attribute membership and conjunction, respectively, and where the domain of values is, e.g.,

$$\text{dom}(S_3) = \underbrace{[0',1000']}_{=A_{3,1}} \times \underbrace{\{\text{degrees of redness in some scale}\}}_{=A_{3,2}} .$$

The symbol $|$ refers to conditioning. (See the next section for further explication.)

Here, S_3 corresponds to the fuzzy set (membership function) $g_3 : \text{dom}(S_3) \to [0,1]$ in the compound form

$$g_3(x,y) = g_{3,1}(x) \odot g_{3,2}(y) \ ; \ x = \text{ht(ship)}, \ y = \text{col(flag)} ,$$

where functions $g_{3,1} : A_{3,1} \to [0,1]$ and $g_{3,2} : A_{3,2} \to [0,1]$ are both obtained from expert prior advice and intelligence information. The range values of the $g_{3,i}$ are possibilities - in general, representing overlapping compound events, and hence not necessarily disjoint probabilities. The operator $\odot$ is not necessarily multiplication and is obtained following the specification of the stochastic interpretation: Each $g_{3,i}$ can be identified with the one point coverage of random set $g_{3,i}^{-1}[U_{3,i},1]$, or equivalently as

$$(U_{3,i} \leq g_{3,i}(x))_{x \varepsilon A_{3,i}} \quad \text{or equivalently} \quad (U_{3,i}^{-1}[0, g_{3,i}(x)])_{x \varepsilon A_{3,i}} .$$

Each $U_{3,i}$ is a random variable uniformly distributed over the unit interval $[0,1]$ and the joint distribution of $U_{3,1}$ and $U_{3,2}$ - as well as with other similarly introduced uniform-$[0,1]$ random variables - i determined by experts or from previous knowledge. In particular, one extreme case is where the $U_{3,i}$ are all identical; another is where they are the negation (unity minus the value) of each other; an intermediate case is where thay are all statistically independent, among an infinity of other possible levels of correlation. All of this corresponds to choices of the operator $\odot$, called a *copula* in the literature ([19],Chapter 2.3.6).

In summary, all C^3 variables can be expressed as states of random variables or in a related form as collections of such descriptions, indexed by the points in the associated domains when the variables are linguistic in nature.

UNCONDITIONAL LOGICAL OPERATORS / RELATIONS

Following the determination of all variables and the appropriate transforms and domains of variables, logical operations are next considered. These are merely formal counterparts for the ordinary set and classical logic operators • (and, conjunction, etc.), v (or, disjunction, etc.), ( )' (not, negation, complement, etc.). As usual, these operators obey the laws of boolean algebra relative to any variab les (or their domain values). Thus, if X,Y,Z are any C^3 variables, provided it is meaningful to apply any of these operators throughout a given relation, one has [20]:

$$X*(Y*Z) = (X*Y)*Z \qquad \text{associativity}, \tag{1}$$

$$X*X = X \qquad \text{idempotency}, \tag{2}$$

$$X*Y = Y*X \qquad \text{commutativity}, \tag{3}$$

for $* = \cdot\, , \mathrm{v}$.

$$0 \,\mathrm{v}\, X = X = \Omega\cdot X \qquad \text{identity}, \tag{4}$$

$$X\cdot(Y \,\mathrm{v}\, Z) = (X\cdot Y) \,\mathrm{v}\, (X\cdot Z)\ ;\ X \,\mathrm{v}\, (Y\cdot Z) = (X \,\mathrm{v}\, Y)\cdot(X \,\mathrm{v}\, Z) \qquad \text{distributivity}, \tag{5}$$

$$(X\cdot Y)' = X'\mathrm{v}Y'\ ;\ (X \,\mathrm{v}\, Y)' = X'\cdot Y' \qquad \text{deMorgan}, \tag{6}$$

$$X'' = X \qquad \text{involution}, \tag{7}$$

$$0' = \Omega\ ,\ \Omega' = 0 \qquad \text{zero-unity properties}, \tag{8}$$

$$X\cdot X' = 0\ ,\ X \,\mathrm{v}\, X' = \Omega \qquad \text{orthocomplementation / law of excluded middle}, \tag{9}$$

$$X \,\mathrm{v}\, (X\cdot Y) = X = X\cdot(X \,\mathrm{v}\, Y) \qquad \text{absorption}, \tag{10}$$

noting that all of the above axioms for boolean relations are not independent of each other, but are presented for purpose of completeness.

In addition, one has the basic partial order (corresponding to subset inclusion)

$$X \leqslant Y \ \text{ iff }\ X = X\cdot Y \ \text{ iff }\ Y = X \,\mathrm{v}\, Y\ , \tag{11}$$

with strict order $<$ (corresponding to proper subset inclusion) holding when $\leq$ holds but $=$ does not, i.e.

$$X < Y \quad \text{iff} \quad X \leq Y \quad \& \quad X \neq Y \, . \tag{12}$$

Finally, unless otherwise indicated, the normalization axiom will be assumed here for all variables of interest:

$$\bigvee_{X \varepsilon \mathrm{dom}(X)} X = \Omega \, , \tag{13}$$

somewhat abusing notation, where in place of the top X and lower left X, technically speaking, one should use a dummy variable denoting a typical possible value of X in dom(X). The above also means that the domain of X is a possibly overlapping but exhaustive covering of Ω.

CONDITIONAL LOGICAL OPERATORS / RELATIONS

While many readers of this work will be familiar with the axioms characterizing boolean algebra of unconditional classical logical relations presented in the previous section, few, if any will recognize the following analogue for *conditional* logical operators and relations, yet such conditioning plays a key role in much of the problems arising in C^3 and elsewhere. Due to historical reasons a gap has existed between conditioning in probability and that in classical logic. In [12,13] this is rectified through the rigorous derivation of a sound and practical to implement, calculus of operators and relations. For example, if one wishes to evaluate the expression $p(s)$, where

s = "if event b occurs then a happens or if d occurs then c" ,

where e.g.

a = "enemy resupplies sector A" , b = "enemy has increased sector C men",

c = "enemy will advance against us", d = "enemy has increased sector B supply" ,

no current standard probability procedure exists for dealing with this which is both mathematically sound and efficient and which is compatible with the usual interpretations

$p(\text{"if b then a"}) = p(a|b)$ $(= p(a \cdot b)/p(b)$, provided $p(b) > 0$),

$p(\text{"if d then c"}) = p(c|d)$.

On the other hand, the new development permits the full evaluation of p(s) as

$$p(s) = p((a\cdot b)v(c\cdot d) | (a\cdot b) \ v \ (c\cdot d) \ v \ b\cdot d)) ,$$

thus obtainable through the usual laws of (unconditional) probability, such as

$$p((a\cdot b)v(c\cdot d)) = p(a\cdot b) + p(c\cdot d) - p(a\cdot b\cdot c\cdot d) ,$$

etc.

The above problem holds because of the appearance of *distinct antecedents in the conditional information.* In any case, the new axioms or laws governing the behavior of conditional events of the form $(X|Y)$, read as "if Y than X", or "X given Y", are for all (unconditional) X,Y,Z,W :

Evaluation: $p((X|Y)) = p(X|Y)$, for all prob. p over events X,Y,W,Z,.. (14)

$(X|\Omega) = X$ extension, $(X|Y) = (X\cdot Y|Y)$ invariance of consequent-to-antecedent, (15)

$(X|Y) = (W|Z)$ iff $X\cdot Y = W\cdot Z$ & $Y = Z$ identification, (16)

$(X|Y)|(W|Z) \approx (X\cdot Y\cdot W\cdot Z \mid Y\cdot((W\cdot Z)v(X'\cdot Z')))$ homomorphic identification of higher order conditioning, (17)

$(X|Y)' = (X'|Y) = (X'\cdot Y|Y)$ negation, (18)

$(X|Y)\cdot(W|Z) = (X\cdot Y\cdot W\cdot Z \mid (X'\cdot Y)v(W'\cdot Z)v(Y\cdot Z))$ conjunction, (19)

$(X|Y)v(W|Z) = ((X\cdot Y)v(W\cdot Z) \mid (X\cdot Y)v(W\cdot Z)v(Y\cdot Z))$ disjunction, (20)

$$(X|Y)\times(W|Z) = (X\times W \mid Y\times Z) = ((X\cdot Y)\times(W\cdot Z) \mid Y\times Z) \tag{21}$$

cartesian product relative to product boolean algebra.

Partial ordering is extended and characterized as

$$(X|Y) \leqslant (W|Z) \text{ iff } (X|Y) = (X|Y)\cdot(W|Z) \text{ iff } (W|Z) = (X|Y) \ v \ (W|Z)$$

$$\text{iff } X\cdot Y \leqslant W\cdot Z \ \& \ W'\cdot Z \leqslant X'\cdot Y , \tag{22}$$

with a similar form for strict order.

All of the above leads to an algebraic structure for the set of all conditional events $(X|Y)$, though not quite boolean, is a relatively pseudocomplemented lattice which is also a Stone algebra with additional properties. (Again, see [12,13] for further properties.)

SOME SPECIFIC SHOR PARADIGM RELATIONS AS AXIOMS

With the general logical structure of variable relations established, the remaining axioms required to specify the formal C^3 theory fully are now given. These relations essentially divide up into two types: weak sufficiency axioms and strong sufficiency axioms. The weak corresponds to the classical sufficiency conditions in probability, and hence are dependent on the specification of particular families of cdf's. For example, when processing information, if the regression relation introduced previously becomes a linear one and *if* noise Q and structure variable T are jointly gaussian distributed , where p indicates the cdf and the regression relation is

$$S = B \cdot R + Q \quad , \tag{f}$$

B a constant m by k real matrix of rank k, Q m by 1, S m by 1, then one has the relation

$$p(T|S) = p(T|\hat{R}) \quad ; \; \hat{R} = (B^T \cdot \mathrm{Cov}^{-1}(Q) \cdot B)^{-1} \cdot B^T \cdot \mathrm{Cov}^{-1}(Q) \cdot S \; . \tag{g}$$

It should be noted also that $\hat{R}$ is the best least squares estimate of R through S which is absolutely unbiased, etc. (See, e.g., [21].)

However, when the above assumptions do not hold, then the corresponding sufficiency condition is invalid. On the other hand, *independent of the probability chosen and the specific function froms involved,* the following strong sufficiency conditions hold relative to being conditional events:

$$(N^{++}|R^{++} \cdot T^{+} \cdot N^{+} \cdot S \cdot R^{-} \cdot N) = (N^{++}|R^{++} \cdot N^{+}) \; , \tag{h}$$

$$(R^{++}|T^{+} \cdot N^{+} \cdot S \cdot R^{-} \cdot N) = (R^{++}|DEC^{+} \cdot N^{+}), \tag{i}$$

$$(T^{+}|N^{+} \cdot S \cdot R^{-} \cdot N) = (T^{+}|N^{+}), \tag{j}$$

etc., where all of the above are derived as reasonable fits to the sequence of data processing occurring within a typical node during the SHOR paradigm. (See Figures 2 and 3.) A longer list of strong sufficiency relations can be found in [17], p. 97. Further subdivisions of variables such as for T ans S can lead to additional relations such as e.g. requiring

$$(DEC|S \cdot DET \cdot HYP \cdot ALG) = (DEC|S) \vee (DEC|ALG) \quad , \tag{k}$$

to reflect possible man-over-ride relative to use of algorithms available for incoming "signal".

THEOREMS DEDUCED FROM THE FORMAL THEORY

In summary, the formal theory of C^3 consists of: the usual alphabet with appropriate sub- and super-scripts to indicate time and node identification ; eqs.(a)-(e),(h)-(j) (with additional axioms representing further subdivisions of relations such as in eq.(k)) representing C^3 proper relations; eqs.(1)-(12) representing the unconditional classical logical operators and relations constituting boolean algebra; eqs.(15)-(22) representing the conditional extension of logical operators and relations; and, finally, the evaluations and interpretations furnished in(12),(14) and (f),(g) when appropriate. In addition, the preliminaries to implementing the theory include the evaluation of specific domains of variables and the replacement of linguistic descriptions by stochastic ones, described in the previous sections.

Next, a simple list of results is presented in the form of Theorems 1-3, leading, in turn, to the chief results- Theorems 4,5 in which the data processing cycle of a typical node according to the SHOR paradigm is quantified recursively.

Theorem 1. Equal antecedent case for combining conditional forms.

For any C^3 variables $X_1,\ldots,X_n,Y$ and logical operators, such as $\cdot$ and v , or any well-defined combination of them, indicated by $*$,

$$(X_1|Y)*\ldots*(X_n|Y) = (X_1*\ldots*X_n|Y)$$

Proof: Use conditional event algebra axioms specialized to the equal antecedent case (see eqs.(18)-(20) with Y=Z).

■

Theorem 2. Conditional forms in expanded disjunctive expressions of auxiliary variables.

For any C^3 variables X,Y initially given and any auxiliary C^3 variables chosen for convenience, say $Z_1,\ldots,Z_n$, assuming normalization of the Z_i,

$$(X|Y) = \bigvee_{\substack{\text{all } Z_i \in \text{dom}(Z_i),\\ i=1,\ldots,m}} (X\cdot Z_1\cdot\ldots\cdot Z_m|Y)$$

Proof: Combine Theorem 1 with normalization (12), associativity (1), and identity (4).

■

Theorem 3. Fundamental chaining relation among conditional forms.

For any C^3 variables $X,Y,Z_1,\ldots,Z_m$, the following relation always holds:

$$(X\cdot Z_1\cdot\ldots\cdot Z_m|Y) = (X|Z_1\cdot\ldots\cdot Z_m\cdot Y)\cdot(Z_1|Z_2\cdot\ldots\cdot Z_m\cdot Y)\cdot\ldots\cdot(Z_{m-1}|Z_m\cdot Y)\cdot(Z_m|Y).$$

Procf: Apply iteratively the conjunction axiom (19) to the right hand side above. ∎

Theorem 4. Formal recursive expansion of evolving node states - simplified form.

$$(N^{++}|N^{+}) = \bigvee_{\substack{\text{all } R^{++}\varepsilon\, \mathrm{dom}(R^{++}),\\ DEC^{+}\varepsilon\, \mathrm{dom}(DEC^{+}),\\ \vdots\\ DET^{+}\varepsilon\, \mathrm{dom}(DET^{+})}} F(N^{++},N^{+};R^{++},DEC^{+},\ldots,DET^{+}) ,$$

where

$$\begin{aligned} F(N^{++},N^{+};R^{++},DEC^{+},\ldots,DET^{+}) = {} & (N^{++}|R^{++}\cdot N^{+})\cdot(R^{++}|DEC^{+}\cdot N^{+})\cdot \\ & (DEC^{+}|FUS^{+}\cdot HYP^{+}\cdot ALG^{+}\cdot DET^{+}\cdot N^{+})\cdot \\ & \vdots \\ & (ALG^{+}|DET^{+}\cdot N^{+})\cdot(DET^{+}|N^{+}) \end{aligned} ,$$

where each of the above factors can be decomposed further where required.

Proof: Combine Theorems 2 and 3 and use e.g. (h)-(j). ∎

Theorem 5. Probability evaluation of evolving node states for SHOR paradigm.

Let p be any probability measure. Then

$$p(N^{++}|N^{+}) = \sum_{\substack{\text{all } R^{++}\varepsilon\, \mathrm{dom}(R^{++}),\\ DEC^{+}\varepsilon\, \mathrm{dom}(DEC^{+}),\\ \vdots\\ DET^{+}\varepsilon\, \mathrm{dom}(DET^{+})}} p(F(N^{++},N^{+};R^{++},DEC^{+},\ldots,DET^{+})) ,$$

where

$$p(F(N^{++},N^{+};R^{++},DEC^{+},..,DET^{+})) = p(N^{++}|R^{++}\cdot N^{+})\cdot p(R^{++}|DEC^{+}\cdot N^{+})\cdot p(DEC^{+}|FUS^{+}\cdot HYP^{+}\cdot ALG^{+}\cdot DET^{+}\cdot N^{+})\cdot \cdots \cdot p(ALG^{+}|DET^{+}\cdot N^{+})\cdot p(DET^{+}|N^{+}) \quad .$$

Proof and Remark: The above follows immediately from use of (14) and th... properties of conditional probabilities. Although Theorem 5 could be ... rather easily as a standard application of the expansion of conditional p...tabilities in terms of summing out auxiliary variables, the point to be made here is that any one of the factors could possibly be expressed not necessarily ... imple chaining form, but rather as a nontrivial logical combination of other te... For example, note that eq.(k) or a related form could be used to evaluate $p(DEC^{+}|FUS^{+}\cdot HYP^{+}\cdot ALG^{+}\cdot DET^{+}\cdot N^{+})$ or a similar situation could arise in the evaluation of the conditional data fusion term, due to the possibly many conditional sources yielding it. ■

IMPLEMENTATION OF THE THEORY AND THE C^3 DESIGN GAME

Applying the above outlined theory to a particular C^3 setting requires specification of all appropriate variables and their possible distributions.

Due to the microscopic nature of the approach, an exponential growth can be expected in general for the computations involved as the number of variables is increased for fidelity of modeling. One technique for possible reduction of this load is outlined in [22], where a combination of an "exact" linearization procedure is utilized with gaussian sum expansions of distributions. Another is the judicious use of key relations and the omission or simplification of other... In [23] Girard outlines such an implementation of theory for a reduced versi... the Naval Outer-Inner Air Battle, where a Blue fighter engages an Orange bo... in the outer zone. The full-scale implementation of this is yet to be dev... which includes modeling of missile launches, counterattacks and maneuve... also [22] for an outline of an implementation scheme related to the O... Air Battle and [5] for the alternative approach of Rubin and Mayk ir...

It is intended that the outputs of the model as developed ... developing a full C^3 design game. Here the adversary and friendl... tified with the possible choices one can make subject to the c... rain, politics, resources, etc., for the functional forms of th...

ditional cdf's that can be chosen among the C^3 variables. A summary of a generic C^3 design game following these ideas is presented in Figure 6 below:

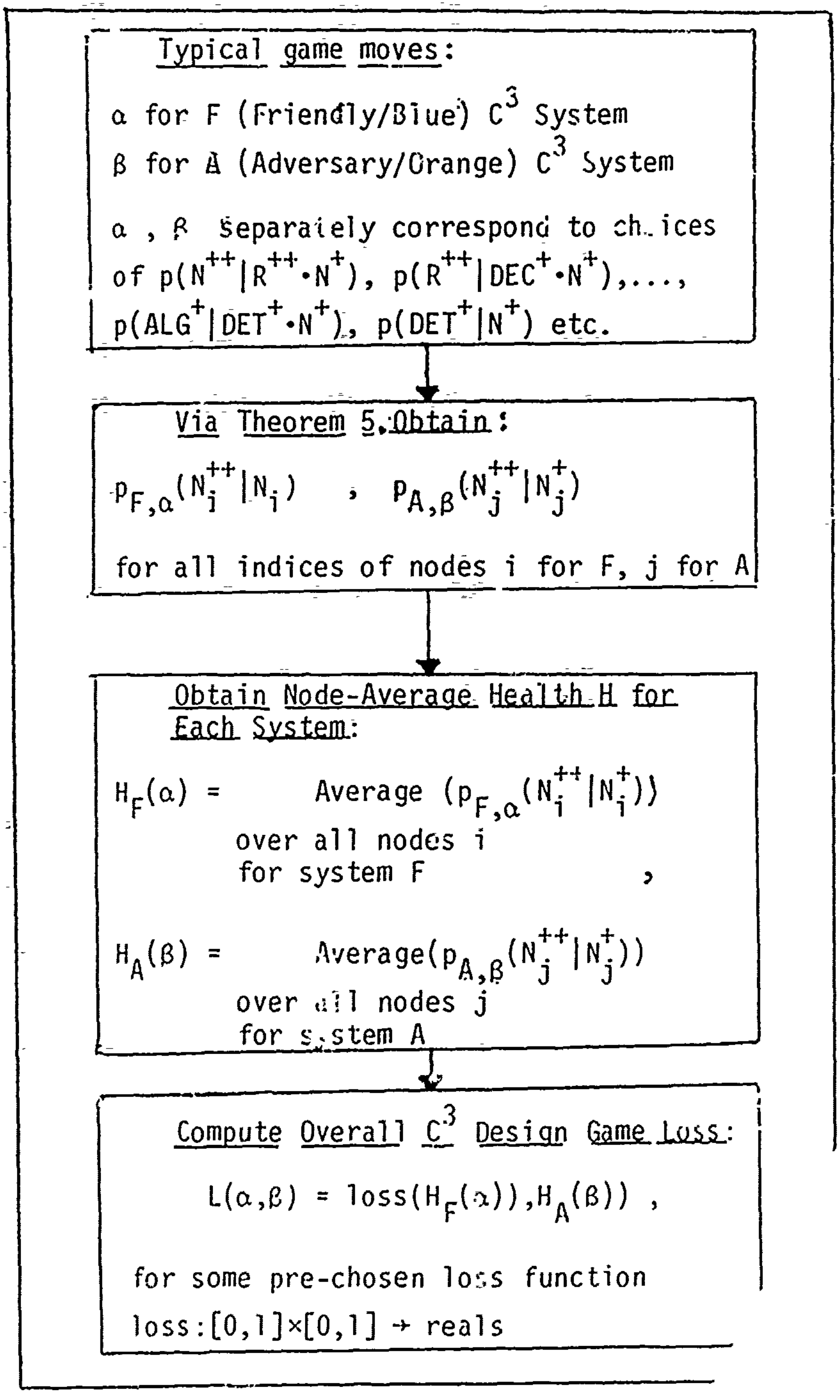


Figure 6. Outline of C^3 Design

REFERENCES

1. Goodman, I.R., "Combination of evidence in C^3 systems", *Proc. 8^{th} MIT/ONR Worksop C^3 Systems*, L.I.D.S, MIT, Cambridge, Mass., Dec.,1985,pp. 161-166.

2. Ingber, L., "Nonlinear, nonequilibrium statistical mechanics approach to C^3 systems", *Proc. 9^{th} MIT/ONR Workshop C^3 Systems*, L.I.D.S., MIT, Cambridge, Mass., Dec., 1986, pp. 237-244.

3. Ingber, L., "C^3 decision aids: statistical mechanics application of biological intelligence", *Proc. 1987 Symp. C^2 Research*, SAIC, McLean,VA, Nov.,1987, pp. 49-57.

4. Rubin, I. & Mayk, I., "Markovian modeling of canonical C^3 systems components", *Proc. 8^{th} MIT/ONR Workshop C^3 Systems*, L.I.D.S., MIT, Cambridge, Mass., Dec., 1985, pp. 15-23.

5. Rubin,I.,Baker,J.,& Mayk,I.,"A stochastic model for the Naval Multi-Phase Outer Air and Inner Air Battles", *Proc. 1989 Symp. C^2 Research*, SAIC, McLean, VA, Nov., 1989, pp. 48-56.

6. Levis, A.H., "Information processing and decisionmaking organizations: a mathematical description", *Proc. 6^{th} MIT/ONR Workshop C^3 Systems*, L.I.D.S., MIT, Cambridge, Mass., Dec., 1983, pp. 30-38.

7. Tomovic, M.M. & Levis, A.H., "On the design of organizational structures for command and control", *Proc. 7^{th} MIT/ONR Workshop C^3 Systems*, L.I.D.S., MIT, Cambridge, Mass., Dec., 1984, pp. 131-138.

8. Cotnier, P. & Levis, A.H., "Assessment of timeliness in command and control", *Proc. 8^{th} MIT/ONR Workshop C^3 Systems*, L.I.D.S., MIT, Cambridge, Mass., Dec., 1985, pp. 39-48.

9. Woodger, J.H., *The Technique of Theory Construction*, in *Encyclopedia of Unified Science, Vol. 2, No. 5*, University of Chicago Press, 1956.

10. Carnap, R., *Introduction to Symbolic Logic and Its Applications*, Dover Publications, New York City, 1958.

11. Kaku, M. & Trainer, J., *Beyond Einstein*, Bantam Books, New York City, 1987.

12. Goodman, I.R., "A measure-free approach to conditioning", *Proc. 3^{rd} AAAI Workshop on Uncertainty in AI*, Univ. of Wash. at Seattle, July, 1987, pp. 270-277.

13. Goodman, I.R., Nguyen, H.T., & Walker, E.A., *Conditional Inference and Logic for Intelligent Systems: A Theory of Measure-Free Conditioning*, monograph to be published by North-Holland Press, Amsterdam, 1991.

14. Goodman, I.R., "A general theory for the fusion of data", *Proc. First Tri-Service Data Fusion Symposium*, A.P.L., Johns Hopkins Univ., Laurel, MD, Dec.,1987, pp. 254-270.

15. Goodman, I.R., "Applications of a conditional event algebra to data fusion", *Proc. 2nd Tri-Service Data Fusion Symp.*, A.P.L., Johns Hopkins Univ., Laurel, MD, Dec., 1988, pp. 179-187.

16. Goodman, I.R., "Applications of a conditional event algebra to data fusion: part 2", *Proc. 3rd Tri-Service Data Fusion Symp.*, A.P.L., Johns Hopkins Univ., Laurel, MD, Dec., 1989. pp. 181-193.

17. Goodman, I.R., "Toward a general theory of C^3 processes", *Proc. 1988 Symp. C^2 Research*, SAIC, McLean, VA, Dec., 1988, pp. 92-105.

18. Goodman, I.R., "Toward a general theory of C^3 processes: part 2", *Proc. 1989 Symposium C^2 Research*, SAIC, McLean, VA, Dec., 1989, pp. 57-67.

19. Goodman, I.R. & Nguyen, H.T., *Uncertainty Models for Knowledge-Based Systems*, North-Holland Press, Amsterdam, 1985.

20. Mendelson, E., *Boolean Algebra and Switching Circuits* (Schaum Outline Series), McGraw Hill Book Co., New York City, 1970.

21. Rao, C.R., *Linear Statistical Inference and Its Applications*, John Wiley Co., New York City, 1973.

22. Goodman, I.R., "Applications of an exact linearization-gaussian sum technique to the modeling of C^3 nodes", *Proc. 1990 Symposium on C^2 Research*, SAIC, McLean, VA, to appear.

23. Girard, P.E . "A combat and decision model based on conditional probability logic", *Proc. 1989 Symp. C^2 Research*, SAIC, McLean, VA, Dec., 1989, pp. 37-47.